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PREDICTION OF WELL LOG INTERPRETATION PARAMETERS INDEPENDENT STUDY

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ABSTRACT

Methods for the prediction of the response of density, acoustic velocity and neutron logs to mixtures of gases, liquids and solids, have been compiled in this paper. The methods are based on knowledge of the elastic constants, chemical make-up, and other physical properties of the material; these data are available from various handbooks or other sources.

Examples are given to illustrate the methods and to demonstrate, with familiar materials, the accuracy of the prediction.

INTRODUCTION

Log analysts recognize that all the various rocks, minerals and fluids surrounding the logging device affect its response. With the advent of modern computer technology as applied to log interpretation, we are now able to take into account many of these variables. It is necessary, therefore, to have an adequate description of these variables, in a form which can be readily understood by both the log analyst and the commonly available log interpretation programs.

This paper is an attempt to lay out, in a systematic manner, techniques which the log analyst

can use to calculate suitable log interpretation parameters from the physical or chemical properties of the rocks and fluids which he feels are likely to be present in the formations traversed by the well logs. The parameters are then used in multiple porosity interpretation methods (i.e., simultaneous solution methods), linear programming techniques (i.e., trial and error methods) or intuitively by the log analyst as he performs a normal log interpretation.

Multiple porosity techniques for quantitative determination of porosity, fluid content and lithology, as well as exploration for hydrocarbons, evaporites, and other non-metallic minerals, are now well established. Several papers covering the field are listed in the references (1, 2, 3, 4, 5, and 6). Techniques using linear programming methods to reduce large quantities of data and constraints have been used with success (7). The use of one or two porosity logs for liquid saturation calculations in gas storage reservoirs has been explained in two papers (8, 9), so the subject is well documented and will not be explained further here.

All the parameters needed can indeed be calculated from the physical properties of the materials. Much data is available in handbooks (14) for this purpose and a large volume of data from many sources was recently compiled privately (15), which could be made available to interested parties on request.

The derivation of the interpretation parameters from these data will be discussed below under the headings of density, velocity and neutron response. Examples are shown, using familiar materials, to illustrate each technique.

DENSITY

Density of Gases

An important concept is the non-ideal gas law: Simply stated it says:

$$\frac{P_1 V_1}{Z_1 T_1} = \frac{P_2 V_2}{Z_2 T_2} \quad \dots (1)$$

where: P = pressure (psia)
 V = volume (cu.ft.)
 Z = gas supercompressibility factor (unitless)
 T = absolute temperature (°R.)

Subscripts 1 and 2 represent data at states 1 and 2, respectively. The concept of a supercompressibility factor, which varies with temperature and pressure, will be used considerably in the derivation of coefficients for various natural gas mixtures.

The composite supercompressibility factor of a mixture is obtained from equation 2:

$$Z_{mix} = \sum_{i=1}^n (Z_i)(Y_i) \quad \dots (2)$$

where: Z_i = supercompressibility factor of the ith component
 Y_i = mole fraction of the ith component
 Z_{mix} = supercompressibility factor of the mixture

Z factors for components of gas can be found in various handbooks. Other methods are available for determination of the Z factor but this is sufficiently accurate for log interpretation purposes. Z factors can also be found in most laboratory fluid analysis reports.

Specific gravity and density are often used interchangeably and this is not correct. Density is weight per unit volume and has units. Specific gravity is the ratio of the weight of a given volume of material to the weight of the same volume of water. In reservoir engineering, density and specific gravity are expressed in several different ways.

The first is the common expression relating weight to volume:

$$\rho = \frac{W}{V} \quad \dots (3)$$

where: ρ = density (lb./cu.ft. or gm./cc.)
 W = weight (pounds or gram)
 V = volume (cu.ft. or cc.)

A useful expression for conversion purposes is:

$$\text{Density in lb./cu.ft.} = 62.4 \text{ times density in gm./cc.} \quad \dots (4)$$

The specific gravity of oil is often given on the API gravity scale and must be converted to density with equation (5).

$$\rho_{oil} = \frac{8829.6}{131.5 + SG_{API}} \quad \dots (5)$$

where: ρ_{oil} = density (lb./cu.ft.)

SG_{API} = specific gravity of oil (API degrees)

The specific gravity of gas is often related to the weight of air, which is defined to be 1.00. Conversion to conventional specific gravity is performed using equation (6).

$$\rho_{gas} = 0.075 \times SG_{GAS} \quad \dots (6)$$

where: ρ_{gas} = density (lb./cu.ft.)

SG_{GAS} = specific gravity referred to air

Densities of mixtures can be calculated from individual densities (in the same fashion as Z factors), by equation (7):

$$\rho_{mix} = \sum_{i=1}^n (\rho_i)(Y_i) \quad \dots (7)$$

where: ρ_{mix} = density of mixture (lb./cu.ft.)

ρ_i = density of the ith component (lb./cu.ft.)

Y_i = mole fraction of the ith component

This applies only to mixtures whose components do not interact, and thus is not the method to use for oil with gas in solution, or chemical compounds. This will be covered later.

Combining equations (1) and (3), for any gas, density is then given by:

$$\rho_{gas} = \frac{PM}{ZRT} \quad \dots (8)$$

where: ρ_{gas} = density (lb./cu.ft.)
 R = gas constant = 10.72 ft./lb. -psi/°R.
 P = pressure (psia)
 Z = gas supercompressibility factor (unitless)
 M = average molecular weight of gas (unitless)
 T = absolute temperature (°R.)

Tables of molecular weights of various hydrocarbon gases can be found in handbooks.

To convert a gas density given at surface conditions to downhole conditions, the following equation is used:

$$\rho_{dh} = 35.37 \times \frac{P_{dh}}{T_{dh} \times Z_{dh}} \times C_{surf} \dots (9)$$

where: ρ = density (lb./cu.ft. or gms/cc)

P = pressure (psia)

T = absolute temperature (°R.)

Z = supercompressibility factor (unitless)

Subscripts "dh" and "surf" represent downhole and surface conditions, respectively.

Standard conditions assumed in these equations are:

$$\begin{aligned} P_{surf} &= 14.65 \text{ psia} \\ T_{surf} &= 520^\circ\text{R.} \\ Z_{surf} &= 1.00 \end{aligned}$$

Graphs of the density of typical gas constituents can be found in handbooks.

The density produced gases can also be found in laboratory fluid analysis reports. Reports for the formation being interpreted, and in the same general area as the well under consideration, can be used for both composition and density of gases. The derivation is shown here in some detail, so that the density of any assumed gas could be computed, if needed. Also, laboratory reports sometimes neglect to state density or Z factors, but never, of course neglect the composition.

An example of the use of some of these concepts is shown below.

Example 1A

Given gas composition:

- C₁H₂ = 51.8% by weight
- C₂H₄ = 28.0% by weight
- C₃H₆ = 20.2% by weight

Find mole fraction of each component -

(1) Component	(2) Weight % (given)	(3) Molec. Weight	(4) Moles/100 lb. (2 ÷ 3)	(5) Mole % (4 ÷ Σ 4)
C ₁ H ₂	51.8	16.04	3.23	70.0
C ₂ H ₄	28.0	30.07	0.92	20.0
C ₃ H ₆	20.2	44.09	0.46	10.0
Total	100.0	-	4.62	100.0

Note: Gas analyses are usually given as mole% so this step is often unnecessary.

Example 1B

Find average molecular weight of this mixture.

$$\begin{aligned} M &= \sum_{i=1}^n (MW_i) (Y_i) \\ &= 16.04 \times 0.70 + 30.07 \times 0.20 + 44.09 \times 0.10 \\ M &= 21.65 \text{ lbs. per lb. mole.} \end{aligned}$$

Example 1C from Ref. (9)

Calculate specific volume, density, supercompressibility factor, and volume occupied at 1,000 psia, 104°F. for 1,000 cu. ft. at standard temperature and pressure, of the gas composition given above; for:

- (1) a perfect gas
- (2) non-ideal gas

1. Treated as a perfect gas having an average molecular weight of 21.65.

(a) Supercompressibility factor of the mixture Z = 1.000 (definition of perfect gas).

(b) Specific volume $v = \frac{RT}{MP} = \frac{10.72 (564)}{21.65 (1,000)} = 0.2795 \text{ cu. ft./lb.}$

(c) Density $= \frac{1}{v} = \frac{MP}{RT} = \frac{21.65 (1,000)}{10.72 (564)} = \frac{1}{0.2795} = 3.578 \text{ lb./cu. ft.}$

2. Treated as a real gas using additive volumes and supercompressibility factors of individual components:

(a) Compressibility factor of the mixture (Z),

$$\begin{aligned} Z &= Z_1(y_1) + Z_2(y_2) + Z_3(y_3) \\ &= 0.918 (0.7) + 0.274 (0.2) + 0.235 (0.1) \\ &= 0.721 \end{aligned}$$

(b) Specific volume $v = \frac{ZRT}{MP} = \frac{0.721(10.72)(564)}{21.65(1,000)} = 0.2015 \text{ cu. ft./lb.}$

(c) Density $= \frac{1}{v} = \frac{MP}{ZRT} = \frac{21.65 (1,000)}{0.721(10.72)(564)} = \frac{1}{0.2015} = 4.963 \text{ lb./cu. ft.}$
(0.079 gm./cc.)

(d) Volume of 1,000 Scf. at 1,000 psia and 104°F.

$$v = \frac{P_s TZ}{P T_s Z_s} V_s = \frac{14.65 (564) (0.721)}{1,000 (520) (1.000)} 1,000 = 11.46 \text{ cu. ft.}$$

Further details of the method can be found in references (10) and (11).

Density of Liquids

Oil densities are almost invariably referred to surface conditions. If stock tank oil gravity can be assumed from nearby wells, this must be first converted to density, using equation (5).

The downhole density is calculated from equation (10):

$$\rho_{dh} = \frac{(GOR \times \rho_{gas} \times 0.178) + \rho_{oil}}{B_o} \dots (10)$$

where: ρ_{dh} = oil density downhole (lb./cu.ft.)
 GOR = solution gas to oil ratio (cu.ft./bbl.)
 ρ_{gas} = solution gas density at surface (lb./cu.ft.)
 ρ_{oil} = stock tank oil density at surface (lb./cu.ft.)
 B_o = formation volume factor of oil and dissolved solution gas (STB./Bbl.)

The various factors in equation (10) must be assumed or taken from fluid analyses from nearby wells. Some average values can be found in handbooks if detailed analyses are unavailable.

Water density can be found from surface measurements and a laboratory measurement of the water formation volume factor or from charts (e.g. Ref. (9), p. 456). The relation is shown in equation (11):

$$\rho_{dh} = \frac{\rho_{surf}}{B_w} \dots (11)$$

where: ρ_{dh} = water density downhole (lb./cu.ft.)
 ρ_{surf} = water density at surface (lb./cu.ft.)
 B_w = water formation volume factor

Example 2A

Find the density of oil with the following specifications:

Temperature = 160°F.
 Pressure = 2,000 psia
 Oil gravity = 38° API
 GOR = 300 cu.ft./bbl.
 ρ_{gas} = 0.002 lb./cu.ft.
 B_o = 0.728

$$\rho_{oil} = \frac{8829.6}{131.5 + 38.0}$$

$$= 52.09 \text{ lb./cu.ft. } (= .834 \text{ gm./cc.})$$

$$\rho_{dh} = \frac{300 \times 0.002 \times 0.178 + 52.09}{0.728}$$

$$= 71.70 \text{ lb./cu.ft. } (=1.148 \text{ gm./cc.})$$

Example 2B

Find the density of water under the same reservoir conditions, having a $B_w = 0.90$, and a salinity of 200,000 ppm. Assume water with this salinity has a density of $1.139 \times 62.4 = 71.0$ lb./cu.ft. (from chart, reference (10)).

$$\rho_{dh} = \frac{71.0}{0.90} = 78.8 \text{ lb./cu.ft. } (=1.246 \text{ gm./cc.})$$

Density of Solids

The density of solid mixtures can be calculated using equation (7), where Y_i is the weight fraction of the i th component. Again, this only applies to mixtures whose components do not react upon combination. In other words, the density of a sand-shale mixture or a sand-water mixture, for instance, can be reconstructed from knowledge of the individual densities, but a breakdown of CaO and CO_2 cannot be reconstituted to calcite ($CaCO_3$) by this method.

Density of compounds can be calculated two ways, as in equations (12) and (13). When the molar volume is known:

$$\rho = \frac{M}{V} \dots (12)$$

where: ρ = density (gm./cc or lb./cu.ft.)

M = moles (gm or lb.)

V = molar volume (cc or cu.ft.)

When the molar volume is unknown but x-ray crystallographic data are available:

$$\rho = \frac{M \times Z}{V_c \times 6.02 \times 10^{23}} \dots (13)$$

where ρ = density (gm/cc or lb./cu.ft.)

M = moles (gm or lb.)

Z = number of formula weights per unit cell (unitless)

V_c = unit cell volume (cc or cu.ft.)

Unit cell data and molar volumes are tabulated in reference (14).

Well logs do not measure true density but respond to the electron density. The electron density of elements seen by the logs is calculated from equation 14.

$$\rho_{log} = \frac{2Z}{A} \times \rho_{true} \dots (14)$$

where ρ_{log} = log density (gm/cc or lb./cu.ft.)
 ρ_{true} = actual density (gm/cc or lb./cu.ft.)
 Z = atomic number (unitless)
 A = atomic weight (unitless)

For compounds or mixtures, the composite Z/A can be calculated from:

$$Z/A = \frac{\sum_{i=1}^n Z_i}{M} \quad \dots (15)$$

where: M = molecular weight of the compound
Z_i = atomic number of the ith element

Note that Z is used to designate two different variables in the foregoing discussion.

Example 3

Calculate density log response to pure salt (NaCl).

$$\rho = \frac{(23 + 35.4) \times 4 \times 62.4}{179.42 \times 10^{-24} \times 6.02 \times 10^{23}}$$

$$= 135 \text{ lb./cu.ft. } (= 2.16 \text{ gm/cc.})$$

$$\rho_{\log} = \frac{2(11 + 17)}{58.4} \times 135$$

$$= 129 \text{ lb./cu.ft. } (= 2.07 \text{ gm/cc})$$

VELOCITY

Sound Velocity in Gases

The velocity of sound in gas is given by:

$$V_{\text{gas}} = 68.4 \sqrt{\frac{KP}{\rho}} \quad \dots (16)$$

where: V_{gas} = velocity of sound (ft./sec.)
K = ratio of specific heats (unitless)
P = pressure (lb./sq.in.)
ρ = density (lb./cu.ft.)

Example 4

Find the velocity of sound in a gas with a same composition and under the same reservoir conditions as Example 1, given the ratio of specific heats equal to 1.37.

$$V_{\text{gas}} = 68.2 \sqrt{\frac{1.37 \times 1,000}{4.963}}$$

$$= 1,150 \text{ ft./sec.}$$

$$\text{Sonic travel time} = \frac{10^6}{1,150} = 885 \text{ u sec./ft.}$$

Sound Velocity in Liquids

Velocity of sound in water or oil can be found from:

$$V_{\text{liq}} = \sqrt{\frac{144}{C_e C_{\text{liq}}}} \quad \dots (17)$$

where: V_{liq} = velocity of sound in liquid (ft./sec.)
g_c = acceleration of gravity = 32.7 ft./sec.²
C_e = compressibility of liquid (psi⁻¹)
ρ_{liq} = density of liquid at downhole conditions (lb./cu.ft.)

C_e usually must be found from laboratory measurements.

Example 5

Find the sound velocity of the oil and water specified in Example 2.

$$V_{\text{oil}} = \sqrt{\frac{144 \times 32.17}{3.0 \times 10^{-6} \times 71.70}}$$

$$= 4,640 \text{ ft./sec.}$$

$$T_{\text{oil}} = 206 \text{ u sec./ft.}$$

$$V_{\text{water}} = \sqrt{\frac{144 \times 32.17}{2.0 \times 10^{-6} \times 78.80}}$$

$$= 5,450 \text{ ft./sec.}$$

$$T_{\text{water}} = 184 \text{ u sec./ft.}$$

Velocity in Solids

The velocity of longitudinal waves in solids can be predicted from equations 18 or 19.

$$V_p = 68.4 \sqrt{\frac{K + 4/3u}{\rho}} \quad \dots (18)$$

$$V_p = 68.4 \sqrt{\frac{E}{\rho \frac{(1-\nu)}{(1-2\nu)}(1+\nu)}} \quad \dots (19)$$

where:

V_p = velocity (ft./sec.)
K = bulk modulus of elasticity (PSI)
u = shear modulus (modulus of rigidity) (PSI)
ρ = density (lb./cu.ft.)
E = Young's modulus (PSI)
ν = Poisson's ratio (unitless)

The elastic constants K, u, E and ν are often known, and many values are listed in reference 14. Identities exist which show that knowledge of any two constants infers knowledge of the other two which, in turn, infers knowledge of velocity. These identities (taken from reference 14) follow and will be collectively known as equation 20.

CONNECTING IDENTITIES FOR ELASTIC CONSTANTS OF ISOTROPIC BODIES

K = bulk modulus; E = Young's modulus; μ = shear modulus;
 β = compressibility = 1/K; λ = Lamé's constant; σ = Poisson's ratio;
 ρ = density; R₁ = V_P/V_S; R₂² = K/(ρV_S²); R₃² = K/(ρV_P²).

K	E	λ	σ	ρV _P ²	ρV _S ² = μ
λ + 2μ/3	μ $\frac{3λ + 2μ}{λ + μ}$..	$\frac{λ}{2(λ + μ)}$	λ + 2μ	..
..	$9K \frac{K - λ}{3K - λ}$..	$\frac{λ}{3K - λ}$	3K - 2λ	3(K - λ)/2
..	$\frac{9Kμ}{3K + μ}$	K - 2μ/3	$\frac{3K - 2μ}{2(3K + μ)}$	K + 4μ/3	..
$\frac{Eμ}{3(3μ - E)}$..	μ $\frac{E - 2μ}{3μ - E}$	E/(2μ) - 1	μ $\frac{4μ - E}{3μ - E}$..
..	..	3K $\frac{3K - E}{9K - E}$	$\frac{3K - E}{6K}$	3K $\frac{3K + E}{9K - E}$	$\frac{3KE}{9K - E}$
λ $\frac{1 + σ}{3σ}$	λ $\frac{(1 + σ)(1 - 2σ)}{σ}$	λ $\frac{1 - σ}{σ}$	λ $\frac{1 - 2σ}{2σ}$
μ $\frac{2(1 + σ)}{3(1 - 2σ)}$	2μ(1 + σ)	μ $\frac{2σ}{1 - 2σ}$..	μ $\frac{2 - 2σ}{1 - 2σ}$..
..	3K(1 - 2σ)	3K $\frac{σ}{1 + σ}$..	3K $\frac{1 - σ}{1 + σ}$	3K $\frac{1 - 2σ}{2 + 2σ}$
$\frac{E}{3(1 - 2σ)}$..	$\frac{Eσ}{(1 + σ)(1 - 2σ)}$..	$\frac{E(1 - σ)}{(1 + σ)(1 - 2σ)}$	$\frac{E}{2 + 2σ}$
ρ(V _P ² - $\frac{5}{3}$ V _S ²)	$\frac{9ρV_S^2 R_2^2}{3R_2^2 + 1}$	ρ(V _P ² - 2V _S ²)	See below.

$$2σ = (R_1^2 - 2)/(R_1^2 - 1) = (3R_2^2 - 2)/(3R_2^2 + 1) = 2(3R_3^2 - 1)/(3R_3^2 + 1).$$

The transverse (shear) wave velocity is defined by equations 21 and 22.

$$V_t = 68.4 \sqrt{\frac{u}{\rho}} \quad \dots (21)$$

$$V_t = 68.4 \sqrt{\frac{E}{\rho} \frac{1}{2(1 + \sigma)}}$$

where: All variables are as defined before.

Considerable data are available on the elastic constants versus pressure. Three methods are available for tabulation of results and all are covered in reference 14.

The first and simplest relates compressibility (which is the inverse of the bulk modulus K) and pressure using equation 23.

$$C_e = \frac{1}{K} = 6.89 \times 10^{-8} a + 47.5 \times 10^{-16} b P \dots (23)$$

where:

- C_e = compressibility (psi⁻¹)
- K = bulk modulus (psi)
- P = pressure (psi)
- a = constant (psi⁻¹)
- b = constant (psi⁻²)

The constants a and b for particular solids can be found in reference 14.

The Voigt and Reuss schemes for obtaining the elastic constants of quasi-isotropic aggregates of a single material lead to the following relationships, where C_{pq} and S_{pq} refer to the individual crystal, and K and G are the bulk modulus and modulus of rigidity, respectively, of the aggregate:

Voigt ... (24)

$$K = (A + 2B)/3$$

$$G = (A - B + 3C)/5$$

with:

$$3A = (C_{11} + C_{22} + C_{33}) \times 14.5 \times 10^6$$

$$3B = (C_{23} + C_{31} + C_{12}) \times 14.5 \times 10^6$$

$$3C = (C_{44} + C_{55} + C_{66}) \times 14.5 \times 10^6$$

Reuss

$$K = 1/(3a + 6b)$$

$$G = 5/(4a - 4b + 3c)$$

$$3a = (S_{11} + S_{22} + S_{33}) \times 6.89 \times 10^{-8}$$

$$3b = (S_{23} + S_{31} + S_{12}) \times 6.89 \times 10^{-8}$$

$$3c = (S_{44} + S_{55} + S_{66}) \times 6.89 \times 10^{-8}$$

Other coefficients for the aggregate may be obtained from K and G by use of the relationships for the isotropic medium given in equation 20.

The C_{pq} and S_{pq} values are obtained from the tables in reference 14. The modulus of rigidity (G) and the shear modulus (u) are synonymous terms.

Example 6

Assume limestone measured values as follows:

$$\begin{aligned} C &= 2.712 \text{ gm/cc} = 170.0 \text{ (lb./cu.ft.)} \\ E &= 0.789 \text{ mb} = 11.42 \times 10^6 \text{ psi} \\ G &= 0.299 \text{ mb} = 4.35 \times 10^6 \text{ psi} \\ \beta &= 0.32 \end{aligned}$$

$$K = \frac{E}{3(1-2\beta)} = \frac{11.42 \times 10^6}{3(102 \times 0.32)} = 10.6 \times 10^6 \text{ psi}$$

$$V_p = \sqrt{\frac{68.4 \times 10.6 \times 10^6 + (4/3) \times 4.35 \times 10^6}{170.0}} = 21,200 \text{ ft./sec.}$$

$$T = \frac{10^6}{21,200} = 47.4 \text{ u sec./ft.}$$

For many rocks, the elastic constants are known, whereas the velocity is not. This is especially true when the effects of pressure and temperature are being considered. It is also clear that given a reasonable set of elastic constants and either a velocity or density log, the other log can be constructed with confidence. This is especially useful in seismography. Note also that the sonic velocity log as a rule measures the travel time associated with the longitudinal wave; therefore, equation 18 or 19 should be used for log interpretation work.

NEUTRON RESPONSE BASED ON HYDROGEN CONCENTRATION

Hydrogen Content of Gases

Since one pound mole of gas contains 2.74×10^{26} molecules, it can be shown that the number of hydrogen atoms in a cubic foot of gas is:

$$h = 1.86 \times 10^{25} \times \frac{HP}{ZT} \quad \dots (25)$$

where:

- h = number of hydrogen atoms per cubic foot of gas
- H = average number of hydrogen atoms per molecule
- Z = gas supercompressibility factor (unitless)
- P and T are as defined before

Example 7

Find the number of hydrogen atoms per cubic foot of the gas from Example 1, held at 2,000 psia and 160°F.

Since the composition was given in Example 1, it can be shown that there are an average of 4.8 atoms of hydrogen per molecule.

$$h = 1.86 \times 10^{25} \times \frac{4.8 \times 2,000}{0.721 \times 620} = 3.99 \times 10^{26} \text{ atoms/cu.ft.}$$

Hydrogen Content of Liquids

The number of hydrogen atoms in water can be found from the fact that the weight of one hydrogen atom is 1.67×10^{-24} grams. The number of hydrogen atoms can be calculated using the relationship:

$$\begin{aligned} h_{\text{water}} &= \frac{454}{1.67 \times 10^{-24}} \times \frac{H}{M} \times C_{\text{water}} \\ &= 2.72 \times 10^{26} \times \frac{H}{M} \times C_{\text{water}} \quad \dots (26) \end{aligned}$$

where:

- h_{water} = hydrogen concentration (atoms/cu.ft.)
- H = average number of hydrogen atoms per molecule
- C_{water} = density of water at downhole conditions (lb./cu.ft.)
- M = average molecular weight

Fresh water at surface conditions thus has $2.72 \times 10^{26} \times \frac{2}{18} \times 62.4 = 18.83 \times 10^{26}$ atoms/cu.ft.

The number of hydrogen atoms in oil can be found from a similar equation.

$$h_{\text{oil}} = 2.72 \times 10^{26} \times \frac{H}{M} \times C_{\text{oil}}$$

Example 8

Find the hydrogen concentration for the oil and water of Example 2.

$$\begin{aligned} h_{\text{water}} &= 2.72 \times 10^{26} \times \frac{2}{18} \times 78.8 \\ &= 23.62 \times 10^{26} \text{ atoms/cu.ft.} \end{aligned}$$

$$\begin{aligned} h_{\text{oil}} &= 2.72 \times 10^{26} \times \frac{2}{16} \times 69.9 = \\ &23.76 \times 10^{26} \text{ atoms/cu.ft.} \end{aligned}$$

Hydrogen Content of Solids

The hydrogen content of solids, such as coal, can be computed from an equation similar to equations 26 and 27.

$$h_{\text{coal}} = 2.72 \times 10^{26} \times \frac{H}{M} \times C_{\text{coal}} \quad \dots (28)$$

Example 9

An average of a number of bituminous coals listed in handbooks shows the following analysis:

Carbon	72.2% by weight
Hydrogen	5.5% by weight
O ₂ , N ₂ , S	22.7% by weight

The empirical chemical formula of this average coal is (roughly) C₆H_{5.5}O, which has a molecular weight of 96. Average density is 1.45 gm./cc.

$$h_{\text{coal}} = 2.72 \times 10^{26} \times \frac{5.5}{96} \times 1.45 \times 62.4 = 14.00 \times 10^{26} \text{ atoms/cu.ft.}$$

Example 10

Carnallite has the chemical formula $KCl \cdot MgCl_2 \cdot 6H_2O$, with a molecular weight of 276 and a density of 1.27. therefore:

$$\rho_{\text{carn}} = 2.72 \times 10^{26} \times \frac{12}{276} \times 1.27 \times 62.7 = 11.60 \times 10^{26} \text{ atoms/cu.ft.}$$

Apparent Neutron Log Porosity

Since fresh water at surface conditions is defined as being 100% porosity and has a hydrogen concentration of 18.83×10^{26} atoms/cu.ft., the apparent porosity of other hydrogen-bearing substances can be calculated by direct proportion.

$$\phi_n = \frac{h}{18.83 \times 10^{26}} \quad \dots (29)$$

where:
 ϕ_n = apparent neutron log porosity (fractional)
 h = Hydrogen concentration (atoms/cu.ft.)
 This assumes that there are no other major constituents which also effect the neutron log (see following section).

Example 11

Find apparent neutron porosity of the gas from Example 7, the oil and water from Example 8 and the coal from Example 9 and carnallite from Example 10.

	h	$\phi_n = h / 18.83 \times 10^{26}$
carnallite	11.60×10^{26}	0.62
coal	14.00×10^{26}	0.74
oil	23.76×10^{26}	1.26
water	23.62×10^{26}	1.25
gas	3.99×10^{26}	0.21

This is a rather empirical approach to the problem, but it does give some idea of the amount of apparent porosity to expect. An attempt was made to verify these calculations by experiment in a large calibration tank but the logging devices are not designed to read in 100% water or oil environments. As a result, no check on these results is available. They are very reasonable, however, Matrix effects will be dealt with in the next section.

NEUTRON RESPONSE BASED ON DIFFUSION LENGTHS

Neutron response in solids and fluids is related to a parameter called the diffusion length (L_d). The apparent limestone porosity of a rock versus diffusion length is plotted in Figure 1, which has been redrawn from reference (12), for a thermal neutron log of average intensity. Data for epithermal neutron logs have not been found in the references so far, at least in the author's library.

The diffusion length of a rock is calculated from equation 30.

$$L_d = \frac{2.02}{\sqrt{\frac{\rho \sum_{i=1}^n \frac{p_i}{M_i} \sum_{j=1}^n \frac{p_j}{M_j}}{\rho \sum_{i=1}^n \frac{p_i}{M_i}}}} \quad \dots (30)$$

where:
 L_d = diffusion length (ft.)
 ρ = density (lb./cu.ft.)
 p_i = weight fraction of the i^{th} component (unitless)
 M_i = molecular weight of the i^{th} component (unitless)
 $\Sigma_{i=1}^n \frac{p_i}{M_i}$ = molecular capture cross section of the i^{th} component (barns)
 $\Sigma_{j=1}^n \frac{p_j}{M_j}$ = scattering cross section of the i^{th} component (barns)

For chemical compounds, the formula is:

$$L_d = \frac{2.02 M}{\sqrt{\sum_{i=1}^n \frac{V_i \Sigma_{i,c}}{M_i} \sum_{j=1}^n \frac{V_j \Sigma_{j,s}}{M_j}}} \quad \dots (31)$$

where:
 V_i = number of atoms of the i^{th} element in the compound (unitless)
 M = molecular weight of the compound (unitless)
 $\Sigma_{i,c}$ = molecular capture cross section of the i^{th} element (barns)
 $\Sigma_{j,s}$ = scattering cross section of the j^{th} element (barns)

And for elements:

$$L_d = \frac{2.02 A}{\sqrt{s}} \quad \dots (32)$$

where:
 A = the atomic number (unitless)
 s = scattering cross section (barns)

Cross section values and summaries of data may be found in reference 12.

Example 12

Calculate the diffusion length of the bituminous coal of Example 9 from its elemental values

	V_i	$\Sigma_{i,c}$	$\Sigma_{i,s}$	V_j	$\Sigma_{j,c}$	$\Sigma_{j,s}$	V_k	$\Sigma_{k,c}$	$\Sigma_{k,s}$
C	6.0	0.0042	4.8	0.0042	0.024	28.8			
H	2.2	0.31	42	0.31	1.7	232			
O	1.0	0.0016	4.1	0.0016	0.0016	4.1			
TOTAL					1.72				268

$$L_d = \frac{2.02 \times 96}{\sqrt{91 \times \sqrt{1.72 \times 268}}} = 0.100 \text{ ft.}$$

$$M = 96$$

$$= 1.42 \text{ gm./cc.} \times 62.4 = 0.1 \text{ lb./cu.ft.}$$

This agrees closely with values given in tables and represent 62% apparent porosity on the chart given earlier in this section. This compares with an

apparent neutron log porosity of 74% found earlier for this coal by the hydrogen concentration method. Since the graph is rather flat in this region, the difference is not too hard to understand.

A Comprehensive Example

An example of simultaneous equation methods using data generated by these methods is shown in Figure 2. This is a solution of sonic, density and sidewall neutron information over a suspected gas zone in the Slave Point Formation in a well in northern Alberta.

The data used is presented in the following table:

ROCKS =	GAS	DCLO	LIME	ANHY	SULP	IRON	H2O
SYMBOLS =	G	D	L	A	S	I	H
DELTA T =	200.0	43.0	47.5	50.0	122.0	65.0	188.0
DENSITY =	0.235	2.870	2.710	2.980	2.030	5.060	1.000
PHI-IMS =	0.500	0.020	0.000	-0.005	-0.030	0.000	1.000

While the example is somewhat simple, it does illustrate the fact that a log interpretation technique can be used in an unconventional manner. One should remember also that the logging devices utilized here are measuring properties of the flushed zone and any gas calculated represent unflushed gas.

CONCLUSIONS

Methods are available for the prediction of sound velocity, density, and neutron log response in various environments which surround boreholes drilled for hydrocarbon or mineral exploration and production. The data necessary for these predictions are available from handbooks; however, considerable manipulation of the information is often necessary prior to use.

Calculations using these methods on familiar materials give answers in close agreement with expected results. It is therefore likely that use can be made of these methods in complex environments. It is to be hoped that this approach will stimulate further research and thought on the subject of the log response to its environment, and on the use of digital computers in the processing of log data.

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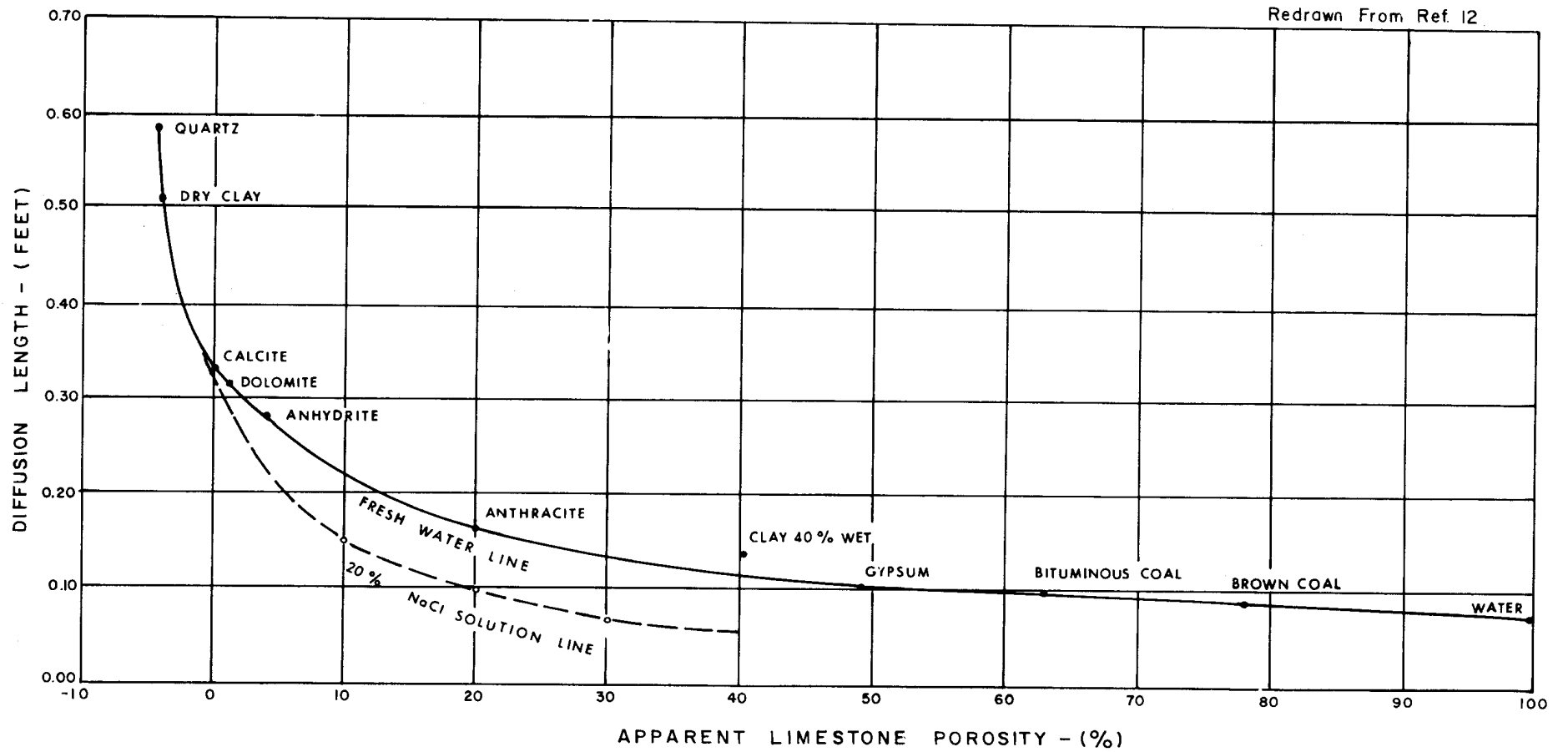
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THERMAL NEUTRON DIFFUSION LENGTH VS. APPARENT LIMESTONE POROSITY



LITHOLOGY POROSITY PLOT
TEST WELL 5

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L I T H O L O G Y L O G

DEPTH	GAS	DOLO	LIME	ANHY	SULP	IRON	H2O
4160	0.4	51.8	39.1	0.0	8.5	0.0	0.0
1	0.5	44.7	46.3	0.0	8.3	0.0	0.0
2	0.7	41.4	50.3	0.0	7.4	0.0	0.0
3	2.0	0.0	94.1	0.0	1.3	2.4	0.0
4	2.0	0.0	94.3	0.0	1.2	2.3	0.0
5	2.0	0.0	94.9	0.0	0.8	2.1	0.0
6	0.0	0.0	91.9	0.0	9.4	0.0	3.5
7	0.6	45.9	44.6	0.0	8.7	0.0	0.0
8	1.1	51.6	39.5	0.0	7.6	0.0	0.0
9	2.4	0.0	94.4	0.0	0.9	2.2	0.0
4170	1.1	41.3	51.3	0.0	6.1	0.0	0.0
1	2.0	0.0	93.5	0.0	1.4	2.9	0.0
2	0.6	63.3	26.2	0.0	9.7	0.0	0.0
3	2.8	0.0	94.9	0.0	0.0	2.2	0.0
4	2.2	0.0	95.2	0.0	1.0	1.4	0.0
5	2.4	0.0	94.0	0.0	1.0	2.4	0.0
6	0.8	51.7	38.8	0.0	8.6	0.0	0.0
7	1.0	44.5	46.9	0.0	7.3	0.0	0.0
8	1.3	47.0	44.8	0.0	6.7	0.0	0.0
9	2.8	0.0	94.4	0.0	0.3	2.3	0.0
4180	0.9	47.1	43.9	0.0	7.9	0.0	0.0
1	0.7	60.7	28.6	0.0	9.8	0.0	0.0
2	2.8	0.0	93.2	0.0	1.4	2.4	0.0
3	0.0	0.0	91.1	0.0	8.5	0.0	3.2
4	0.0	0.0	92.3	0.0	6.4	0.0	2.2
5	3.0	0.0	94.9	0.0	0.5	1.4	0.0
6	2.5	0.0	95.2	0.0	-0.0	2.2	0.0
7	2.0	0.0	96.0	0.0	0.7	1.0	0.0
8	2.0	0.0	96.0	0.0	0.6	1.3	0.0
9	0.2	53.8	38.6	0.0	7.2	0.0	0.0
4190	0.0	62.2	29.0	0.0	8.6	0.0	0.0
1	2.0	0.0	94.9	0.0	0.9	2.0	0.0
2	2.1	0.0	95.7	0.0	-0.0	2.1	0.0
3	1.1	33.0	61.3	0.0	4.4	0.0	0.0
4	2.0	0.0	95.3	0.0	0.6	1.9	0.0
5	1.6	36.7	56.8	0.0	4.8	0.0	0.0
6	1.3	47.0	44.2	0.0	7.3	0.0	0.0
7	0.0	0.0	90.4	0.0	9.6	0.0	4.2
8	4.8	0.0	93.3	0.0	0.2	1.6	0.0
9	1.3	0.0	92.9	0.0	3.6	0.0	2.0
4200	4.4	17.2	76.3	0.0	2.0	0.0	0.0
1	5.5	14.3	78.9	0.0	1.2	0.0	0.0
2	1.3	0.0	92.0	0.0	3.6	0.0	2.9
3	0.0	0.0	91.3	0.0	6.8	0.0	3.8
4	0.0	0.0	90.7	0.0	8.5	0.0	3.6
5	1.5	0.0	94.8	0.0	2.3	0.0	1.3
6	5.4	18.8	74.7	0.0	0.8	0.0	0.0
7	10.9	66.1	24.2	0.0	0.0	-1.3	0.0
8	9.2	0.0	85.6	0.0	-0.9	0.0	6.0
9	8.5	0.0	86.2	0.0	0.0	-0.9	6.1